



System of Linear Equations: A Comparative Study Between Cramer's Rule and Paravartya's Rule

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Background and Aims: Solving systems of linear equations is essential in mathematics education, influencing real-world problem-solving and advanced applications. This study compares the effectiveness of Cramer's rule and Paravartya's rule in enhancing Grade 8 students' achievement in linear equations, aiming to address challenges in teaching methods.

Methodology: A mixed-methods approach was used, combining a quasi-experimental pretest-posttest design with a sequential exploratory design for qualitative data. Quantitative data were obtained through assessments, and qualitative data were collected from student feedback and analyzed thematically.

Results: Findings revealed statistically significant improvements in students' mathematics achievement from pretest to posttest using both methods ($p < 0.05$). Pretest scores were relatively low ($M_c = 28.83$, $M_p = 29.03$), but posttest scores showed notable improvements. Despite the absence of statistically significant differences in posttest outcomes between the two methods ($p = 0.613$), both approaches proved effective in enhancing problem-solving abilities. Key challenges identified included difficulties with signed numbers, large computations, equation transformation, and procedural steps. Students employed coping strategies such as memorization, chunking, consistent practice, and peer support. A Learning Activity Sheet (LAS) was developed to reinforce learning, promote active engagement, and accommodate diverse learning styles.

Conclusion: The study concludes that both Cramer's rule and Paravartya's rule effectively enhance students' performance in solving systems of linear equations. Challenges related to procedural complexity were mitigated through structured interventions, peer collaboration, and practice. The findings highlight the need for targeted instructional strategies and the development of supportive resources, like LAS, to promote mathematical cognition and active learning.

Keywords: Mathematical cognition, Pedagogical innovation, Active learning

Introduction

Teaching Mathematics is a dynamic and essential component of education, fostering problem-solving skills, logical reasoning, and a deeper understanding of mathematical concepts. Mathematics education plays a pivotal role in equipping students with the tools necessary to analyze and address complex problems encountered in academic and real-world contexts. Within this framework, the teaching of linear equations holds particular significance as it introduces students to foundational concepts in Algebra, forming the basis for understanding relationships between variables and supporting applications across disciplines such as economics, engineering, and environmental science (Larson & Edwards, 2013).

Teaching practices are streamlined to emphasize relevance, focus, and essentiality in learning in alignment with the Most Essential Learning Competencies (MELCs) curriculum mandated by the Department of Education. MELCs prioritize critical competencies that foster lifelong learning skills, addressing both academic and real-world needs. While these competencies guide curriculum design and instructional delivery, there remains a need to explore innovative strategies that align with MELCs and enhance student engagement and mastery.

Despite the importance of linear equations, students often encounter significant challenges with traditional methods of solving them, such as substitution, elimination, and graphing. These methods, while effective, can lead to cognitive overload, procedural complexity, and a lack of conceptual understanding, especially for learners who struggle with abstract reasoning. Addressing these challenges underscores the necessity of exploring alternative approaches that simplify processes while strengthening conceptual comprehension.





The study proposes integrating Cramer's rule and Paravartya's rule as alternative methods for solving systems of linear equations. These techniques offer computational efficiency and conceptual clarity, providing students with diverse tools to tackle linear equations effectively. Cramer's rule, grounded in matrix determinants, and Paravartya's rule, rooted in ancient mathematical principles, have the potential to transform instructional practices by addressing students' difficulties with traditional approaches and enhancing their problem-solving skills (Mervat Mohammad et al., 2023; Shankar, 2016).

The theoretical foundation of this study is guided by APOS (Action, Process, Object, Schema) theory, which provides insights into how students develop mathematical understanding. APOS theory emphasizes the importance of moving from actions to processes and eventually forming coherent mental structures (Dubinsky & McDonald, 2001). Introducing APOS theory establishes its relevance in analyzing students' learning processes as they engage with linear equations. By applying this framework, the study seeks to evaluate how alternative methods influence students' cognitive development and mathematical reasoning.

Educationally, integrating Cramer's and Paravartya's rules into the curriculum has far-reaching implications. These methods not only address procedural inefficiencies but also promote analytical thinking and flexibility in mathematical problem-solving. Prior studies have shown that alternative strategies can enhance student performance and confidence, ultimately contributing to improved outcomes and greater interest in mathematics (Mervat Mohammad et al., 2023). Highlighting these implications supports the case for adopting diverse instructional techniques that align with modern educational goals.

By addressing the limitations of traditional approaches, grounding the research in APOS theory, and emphasizing the educational benefits of innovative strategies, the study seeks to contribute to the advancement of mathematics education. This exploration aims to enhance students' conceptual understanding, procedural fluency, and problem-solving abilities, thereby better preparing them for future mathematical challenges.

Objectives

This study aimed to 1) assess students' pretest and posttest performance to measure learning improvement, 2) identify challenges encountered by students when applying these mathematical methods, 3) explore strategies employed by students to overcome these challenges, and 4) develop a learning activity sheet focused on solving systems of linear equations using Cramer's rule and Paravartya's rule to further enhance students' mathematical skills.

Literature review

Cramer's Rule and Paravartya's Rule

Cramer's rule provides a systematic technique for solving systems of linear equations with a unique solution. It represents each variable as a fraction of two determinants: one derived from replacing a single column of the coefficient matrix with the constants from the equations, and the other being the determinant of the original coefficient matrix (Lay, 2012). This approach is computationally efficient for small systems but becomes impractical for larger systems due to the computational complexity of calculating determinants. Paravartya's rule, on the other hand, originates from Vedic Mathematics and was introduced by Tirthaji (1965). It simplifies calculations through specific algebraic manipulations and transformations, offering an alternative to Cramer's rule by focusing on row and column operations within the coefficient matrix (Shrawankar & Sapkal, 2017). This method reduces computational steps and minimizes errors, making it especially useful for students grappling with procedural complexity.

From a theoretical perspective, these methods align with constructivist principles, emphasizing active learning and conceptual understanding. Constructivist theory suggests that students build knowledge through experience and reflection, making methods like Cramer's and Paravartya's rules particularly effective for fostering deeper conceptual understanding. They also address cognitive load theory (Sweller, 1988) by reducing the number of steps required to reach a solution, thereby minimizing extraneous cognitive load and allowing students to focus on meaningful problem-solving processes.





Sweller's theory highlights those instructional strategies should manage cognitive load to optimize learning outcomes, a principle that supports the integration of these methods into teaching practices to enhance conceptual clarity and procedural fluency.

While prior research highlights the benefits of these methods, there are limitations worth noting. Cramer's rule is restricted to systems with unique solutions, limiting its applicability in cases involving infinite or no solutions (Lay, 2012). Similarly, the scalability of Paravartya's rule for larger systems remains underexplored (Shrawankar & Sapkal, 2017). Addressing these gaps can help refine their implementation and identify contexts where they are most effective. Comparative studies evaluating these methods against traditional techniques, such as substitution, elimination, and graphing, underscore their potential advantages in simplifying complex calculations.

This study also examines how these alternative methods address common learning challenges, such as difficulties in transforming equations into standard form and performing calculations with large numbers. By providing structured approaches that reduce procedural complexity, these methods aim to build confidence and reduce frustration among students, particularly those who struggle with traditional methods (Mervat Mohammad et al., 2023). Mohammad et al. (2023) emphasize the importance of addressing procedural difficulties to improve problem-solving efficiency and conceptual understanding, aligning well with the goals of this study.

Recent advancements in digital tools and simulations further support the integration of these methods. For instance, interactive software and visualization tools can complement Cramer's and Paravartya's rules, enhancing students' engagement and conceptual understanding (Sharma, 2021). Sharma (2021) discusses the role of technology-enhanced learning environments in fostering interactive and dynamic learning experiences that reinforce mathematical concepts and problem-solving strategies. Incorporating technology aligns with contemporary educational practices and provides opportunities for differentiated instruction, catering to diverse learning needs.

The study aligns with the Most Essential Learning Competencies (MELCs) curriculum, emphasizing critical thinking, problem-solving, and lifelong learning. The Department of Education highlights the importance of streamlining curriculum content to focus on essential skills that prepare students for real-world applications. Cramer's and Paravartya's rules support these goals by offering students alternative problem-solving strategies that enhance analytical thinking and conceptual comprehension. Their inclusion in the curriculum represents a shift toward competency-based education, ensuring relevance and applicability in addressing real-world problems.

In addressing existing gaps in the literature, this study focuses on Grade 8 students in a Philippine educational context, contributing to global discussions on Mathematics education. It evaluates the practical implementation of these methods within the MELCs framework, providing insights into their effectiveness and scalability. By highlighting the challenges and benefits associated with these approaches, the study offers educators innovative strategies to improve teaching practices and enhance student outcomes.

Ultimately, this research aims to enrich the teaching of systems of linear equations by investigating the pedagogical potential of Cramer's and Paravartya's rules. Through a theoretical foundation grounded in constructivism, cognitive load theory, and APOS theory, it seeks to promote conceptual understanding, procedural efficiency, and problem-solving skills. Dubinsky and McDonald (2001) emphasize that APOS theory provides a structured approach to understanding students' mathematical development through actions, processes, objects, and schemas. This exploration contributes to advancing Mathematics education by providing teachers with practical tools and strategies to foster deeper learning and academic success.

Conceptual Framework

The conceptual framework for this study integrates the challenges, strategies, and educational implications associated with the teaching and learning of systems of linear equations. This study is anchored on the theory of Action-Process-Object-Schema (APOS), which provides a structured lens for analyzing the progression of mathematical understanding. The resolution of systems of linear equations is often perceived as complex due to the diverse strategies required, with students typically employing



traditional methods such as substitution, elimination, and graphing (Mervant Mohammad et al., 2023; Larson & Edwards, 2013). However, innovative techniques such as Cramer's rule and Paravartya's rule offer underutilized yet effective alternatives to simplify these systems.

Paravartya's rule, derived from Vedic Mathematics, reconfigures coefficient matrices to streamline calculations, making it especially advantageous for complex systems (Shrawankar & Sapkal, 2017; Rao & Rangarajan, 2019). Conversely, Cramer's rule uses determinants for systematic computation, making it particularly efficient for smaller systems with unique solutions (Lay, 2012; Zhang & Li, 2018). These alternative methods diversify instructional approaches and offer opportunities to deepen students' conceptual understanding of linear equations.

The APOS theory evaluates how learners interact with these methods by delineating a progression from actions to processes, objects, and schemata (Dubinsky et al., 2005). This hierarchical framework provides insights into students' learning processes, emphasizing the need to move beyond procedural reliance to achieve a more integrated conceptual understanding. Challenges in linear algebra, such as over-dependence on procedural methods and difficulties in conceptualizing mathematical objects, underscore the importance of guided interventions to support learners' transition through the APOS stages (Ndlovu & Brijlall, 2016; Tatira, 2023).

For instance, Paravartya's rule requires students to transform matrices, reinforcing their understanding of algebraic structures and matrix operations as they progress from procedural reliance (Action) to conceptual mastery (Schema). Similarly, Cramer's rule encourages students to interpret determinants geometrically, fostering connections between algebraic and spatial reasoning (Zhang & Li, 2018). These methods complement the APOS framework by providing distinct pathways for developing mathematical schemas. Educational implications of applying APOS theory include addressing procedural difficulties and enhancing conceptual learning. The framework supports interventions that target specific stages of learning, such as guided practice to strengthen actions and processes or exploratory tasks to promote object and schema development. By scaffolding students' progression through APOS stages, these methods can mitigate cognitive overload (Sweller, 1988) and improve problem-solving efficiency. Moreover, the framework highlights how alternative methods can address common errors, such as computational mistakes and difficulties with abstract reasoning. For example, Paravartya's rule minimizes errors by simplifying transformations, while Cramer's rule provides structured algorithms to verify solutions (Shrawankar & Sapkal, 2017; Sharma, 2021). Integrating these approaches with APOS theory thus aligns with curriculum goals to develop critical thinking and analytical skills.

This study also justifies the selection of APOS theory by demonstrating its effectiveness in analyzing learning progressions in Mathematics education. Previous research has successfully applied APOS theory to topics such as functions and algebraic structures, showing its adaptability to varied mathematical contexts (Dubinsky et al., 2005; Ndlovu & Brijlall, 2016). By extending this application to systems of linear equations, the study provides a new perspective on teaching methodologies and learning outcomes. Potential limitations of APOS theory include its reliance on structured instructional designs and its applicability to diverse learner profiles. Variations in students' prior knowledge and cognitive abilities may affect their progression through APOS stages, necessitating differentiated instruction and additional scaffolding. Addressing these constraints, this study incorporates technology-based tools and collaborative learning strategies to support diverse learners.

Finally, the schematic diagram accompanying this framework visually represents the study's structure, showing the relationship between APOS stages, mathematical methods, and learning outcomes. Annotations and explanations clarify how each component aligns with the study's objectives, reinforcing the theoretical and practical connections.

Applying APOS theory to this study involves analyzing students' progression through each stage while using Cramer's and Paravartya's rules. At the Action stage, students execute step-by-step procedures, such as calculating determinants in Cramer's rule or performing matrix manipulations in Paravartya's rule. Moving to the Process stage, students begin to internalize these steps, recognizing patterns and relationships within the computations (Dubinsky, 1991). The Object stage emerges when students conceptualize the entire method as a unified process rather than discrete steps, enabling flexible

application to different problems. Finally, at the Schema stage, learners integrate these conceptual understandings into broader mathematical frameworks, allowing them to generalize and transfer knowledge to novel contexts.

The schematic diagram below outlines a mixed-methods sequential explanatory design combining quantitative and qualitative data. It compares two teaching methods—Cramer’s Rule and Paravartya’s Rule—through pre-tests, treatments, and post-tests. Learning Activity Sheets (LAS) reinforce learning, and qualitative data clarify quantitative results, assessing each method’s effectiveness in teaching mathematical concepts.

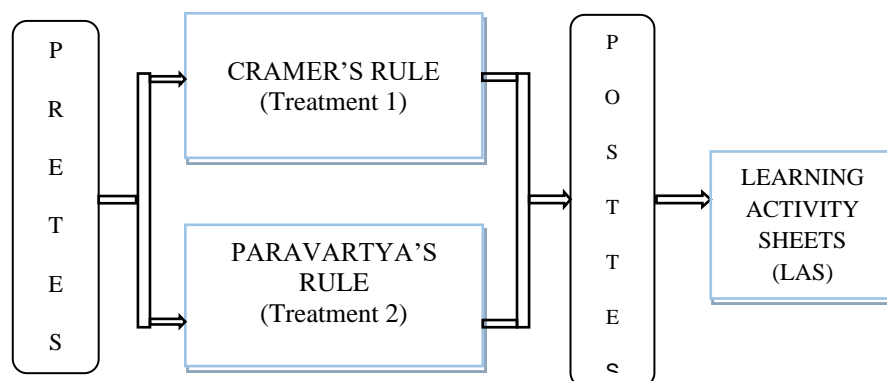


Figure 1. The Schematic Diagram of the Study

Methodology

This study employed a mixed-methods sequential explanatory design, carried out in two phases. The first phase involved collecting and analyzing quantitative data, followed by the collection of qualitative data in the second phase to provide deeper insights and contextualize the quantitative findings (Creswell & Plano Clark, 2018). This design was chosen because it allows the quantitative data to provide a broad understanding of the impact of the methods used to solve linear equations, while the qualitative data offers nuanced insights into students' experiences and challenges, helping to explain the underlying reasons behind the observed quantitative results.

Quantitative Phase

The quantitative phase followed a quasi-experimental pretest-posttest design. This design was chosen over randomized controlled trials (RCTs) due to practical constraints, such as the limited number of sections available and the specific classroom settings, which made random assignment infeasible. The quasi-experimental design allowed for a meaningful comparison of two distinct teaching methods, Paravartya’s Rule and Cramer’s Rule, within naturally occurring classroom groups. Pretests and posttests were used to assess students' competencies in solving linear equations, which are aligned with the Grade 8 MELCs.

The pretest and posttest were designed as validated 30-item assessments, with content validation conducted by four experts. Reliability was confirmed with a Cronbach’s alpha of 0.74 for the pretest and posttest. The Learning Activity Sheets (LAS) for both methods were also validated to ensure they aligned with the learning objectives and the assessment tools. Descriptive statistics (mean and standard deviation) were used to assess students' mastery of linear equations, and ANCOVA was applied to compare the posttest scores while controlling for pretest scores. This analysis was conducted using IBM SPSS Statistics 27. The choice of ANCOVA was based on its ability to control for potential baseline differences between the two groups, thus ensuring a more accurate comparison of the effectiveness of the methods. Assumptions, such as homogeneity of regression slopes, were tested before analysis to ensure the validity of the ANCOVA results.

Qualitative Phase

The qualitative phase involved in-depth interviews with 10 students, selected purposively from the 41 participants. These interviews aimed to provide a deeper understanding of the students' experiences and challenges with the two methods of solving linear equations. The qualitative data served to contextualize and explain the quantitative findings, revealing patterns, insights, and personal reflections that may not have been captured through the pretest and posttest alone.

Thematic analysis (Braun & Clarke, 2006) was used to identify key themes in the qualitative data. To ensure the reliability and rigor of the analysis, inter-rater reliability was established by having two independent coders analyze the interview transcripts. Any discrepancies in coding were resolved through discussion and consensus. This step helped mitigate potential biases and ensured that the themes were consistently identified and accurately represented.

Sampling and Participant Selection

The study utilized purposive sampling to select 41 Grade 8 students from a public school in Ozamiz City, Misamis Occidental, Philippines. These students were aged 13 to 14 and had prior Mathematics grades ranging from 75 to 85, ensuring comparability in academic background. Two sections were selected: 21 students used Paravartya's Rule, and 20 used Cramer's Rule. The sample size of 41 was deemed adequate to detect meaningful differences in performance based on prior research and statistical power analyses. Additionally, the demographic and academic characteristics of the participants were considered in the selection process to ensure that the findings could be generalized within similar educational contexts.

Ethical Considerations

Ethical procedures followed during the study included obtaining formal approval from the Schools Division Superintendent and Principal. Parental consent and student assent were secured before participation. Confidentiality and privacy were maintained throughout the study, particularly during interviews and assessments. Data were anonymized and stored securely to protect participants' identities. The study adhered to ethical standards, ensuring that no harm came to the participants, and plagiarism checks were conducted to maintain academic integrity. The relevant ethical review board approved all procedures to ensure compliance with ethical guidelines for research involving minors.

Statistical and Qualitative Analysis

For qualitative analysis, 10 students provided detailed insights during interviews. A validated 30-item pretest and posttest assessed competencies in solving linear equations aligned with Grade 8 MELCs. Four experts conducted content validation, and Cronbach's alpha (0.74) confirmed reliability. The Learning Activity Sheet (LAS) for both methods was validated alongside the tests. Formal approval was obtained from the Schools Division Superintendent and Principal. Parental consent ensured ethical participation.

Below is the mastery/achievement level used in this study:

MASTERY/ACHIEVEMENT LEVEL	
MPS	Descriptive Equivalent
96 – 100%	Mastered
86 – 95%	Closely Approximating Mastery
66 – 85%	Moving Towards Mastery
35 – 65%	Average
15 – 34%	Low
5 – 14%	Very Low
0 – 4%	Absolutely No Mastery



Descriptive statistics calculated means and standard deviations to assess Mathematics achievement. ANCOVA tested the significance of posttest scores with pretest scores as covariates, analyzed using IBM SPSS Statistics 27. Thematic Content Analysis (Braun & Clarke, 2006) identified qualitative themes. Ethical guidelines ensured validity and reliability.

Results

Levels of Students' Mathematics Achievement

Table 1: Level of Students' Mathematics Achievement Using Cramer's and Paravartya's Rules

Pretest	Mean	SD	MPS	Interpretation
Cramer's Rule	8.65	3.45	28.83	Low
Paravartya's Rule	8.71	3.30	29.03	Low
Posttest				
Cramer's Rule	14.45	3.94	48.17	Average
Paravartya's Rule	14.05	2.82	46.83	Average

Table 1 above shows that based on the pretest and posttest scores, both groups who were taught using Cramer's rule and Paravartya's rule have low Mathematics achievement levels ($M_C = 28.83$, $M_P = 29.03$). Furthermore, the posttest results revealed that both groups of students have average Mathematics achievement levels ($M_C = 48.17$; $M_P = 46.83$). The percentage increase in mean scores was approximately 67% for Cramer's Rule and 61% for Paravartya's Rule, indicating notable gains for both methods.

These results suggest that integrating Cramer's and Paravartya's rules into instruction significantly enhanced students' problem-solving abilities and conceptual understanding. The increase in scores highlights the effectiveness of the instructional approaches, supporting the aim of evaluating their impact on Mathematics achievement.

These findings align with the study of Hamid and Kamarudin (2021), who state that incorporating creative and innovative teaching strategies plays a pivotal role in enhancing students' higher-order thinking skills and overall performance in Mathematics. This underscores the importance of exploring and implementing varied approaches to teaching mathematical concepts to foster deeper understanding and improved academic achievement.

Significant Difference Between the Students' Mathematics Achievement in the Pretest and Posttest Using Cramer's Rule and Paravartya's Rule

Table 2. Test of Significant Difference Between the Students' Mathematics Achievement in the Pretest and Posttest Using Cramer's Rule

	t- value	df	p-value	Decision	Interpretation
Pretest Posttest	- 7.971	19	0.001	Reject H_0	Significant Difference

Table 2 reveals a significant difference in students' Mathematics achievement based on pretest and posttest scores using Cramer's rule, with a $p < 0.05$ confirming its effectiveness as a teaching strategy. This finding highlights the impact of Cramer's rule in enhancing students' understanding and achievement in Mathematics.

The structured, step-by-step approach of Cramer's rule not only simplifies the comprehension of Algebraic concepts but also reduces cognitive load, thereby boosting students' confidence in problem-solving (Johnson & Sharma, 2020; Burns & Martin, 2018). Additionally, it is a valuable tool for addressing math anxiety by providing a clear and manageable framework that fosters mathematical proficiency while minimizing student apprehension (Lee & Yoon, 2019; Raghavendra, 2019). These benefits underscore the effectiveness of Cramer's rule as an innovative teaching strategy for improving learners' overall mathematical competence.





Table 3. Test of Significant Difference Between the Students' Mathematics Achievement in the Pretest and Posttest Using Paravartya's Rule

	t- value	df	p-value	Decision	Interpretation
Pretest Posttest	- 8.623	20	0.001	Reject H ₀	Significant Difference

The results from Table 3 show a significant improvement in students' pretest and posttest scores when using Paravartya's rule ($p < 0.05$), indicating the method's effectiveness in teaching and learning. This observed improvement suggests that Paravartya's rule, as a part of Vedic Mathematics, plays a pivotal role in enhancing students' Mathematical achievement. Paravartya's rule, a technique from Vedic Mathematics, simplifies solving systems of linear equations, making it more approachable and engaging for students. Research supports the effectiveness of such Vedic methods, emphasizing their ability to improve the speed and accuracy of mental calculations while deepening students' conceptual understanding. Agrawal and Gupta (2018) argue that Vedic strategies facilitate a more intuitive grasp of complex Algebraic concepts, enhancing students' Mathematical abilities. Williams (2002) also notes that these techniques expedite problem-solving by streamlining the computational process, making mathematics more accessible and efficient.

Moreover, Vedic Mathematics approaches, such as Paravartya's rule, have been associated with reduced math anxiety and increased student engagement. According to Mehta and Jain (2021), these methods boost student confidence, improve problem-solving abilities, and contribute to better post-test performance. The incorporation of Vedic techniques promotes active learning. It fosters a positive attitude toward Mathematics, reinforcing that Paravartya's rule significantly improves Grade 8 students' Mathematical achievements.

Analysis of Covariance for the Significant Difference Between the Students' Mathematics Achievement in the Posttests Using Cramer's Rule and Paravartya's Rule

Table 4. Analysis of Covariance for the Significant Difference Between the Students' Mathematics Achievement in the Posttests Using Cramer's Rule and Paravartya's Rule

Dependent Variable: Posttest						
Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	163.646 ^a	2	81.823	10.651	.000	.359
Intercept	418.718	1	418.718	54.507	.000	.589
Pretest	161.987	1	161.987	21.087	.000	.357
Method	1.994	1	1.994	.260	.613	.007
Error	291.915	38	7.682			
Total	8774.000	41				
Corrected Total	455.561	40				

a. $R^2 = .359$ (Adjusted $R^2 = .325$)

Table 4 shows the test of significant difference between the students' Mathematics achievement in the posttests using Cramer's rule and Paravartya's rule. The table above showcases the Tests of Between-Subjects Effects of the Analysis of Covariance to compare the means of the post-test scores of Cramer's rule and Paravartya's rule while controlling for the effects of the pretest scores as covariates that might influence the post-test scores. As shown, there is no significant difference between the posttest scores of Cramer's rule and Paravartya's rule, as confirmed in the p-value ($p = 0.613 > \alpha = 0.05$) associated with a very small F – F-value ($F = 0.260$). This would further mean that both methods are equally effective in solving systems of linear equations. Hence, the null hypothesis cannot be rejected, inferring an insignificant difference between the two methods.

Moreover, the analysis resulted in a very small effect size ($\text{Partial Eta Squared} = 0.70\%$), indicating a very weak relationship between the variables. A Partial Eta Squared of 0.70% (or 0.007) means that the factor (method used) explains just 0.7% of the variance in the dependent variable – posttest scores. This is generally interpreted as a *very small effect*, suggesting that while there is a

statistical association between the variables, the factor minimizes the changes in the outcome. In addition, R-squared ($R^2 = 0.359$) indicates that only 35.90% of the total variance in the post-test scores can be accounted for or explained by the two methods, indicating a better fit of the model to the data. This result further implies that both methods significantly improved the students' Mathematics achievement and are equally effective in solving systems of linear equations.

In the context of APOS theory, this study shows how students progress from performing basic actions to understanding and applying mathematical methods as structured processes and objects. Both Cramer's rule and Paravartya's rule helped students improve their mathematics achievement, transitioning from the action stage (basic operations) to the process stage (applying procedures).

These findings align with prior research, suggesting that both Cramer's rule and Paravartya's rule are effective teaching tools for solving linear equations, with neither method proving definitively superior to the other. This conclusion is consistent with studies by Johnson and Sharma (2020), who praised Cramer's rule for enhancing logical reasoning, and Bhargava and Kumar (2019), who highlighted Paravartya's rule's potential to alleviate math anxiety and improve motivation. The results underscore the importance of adaptable teaching methods and suggest that integrating both traditional and innovative strategies can be beneficial in meeting students' diverse needs and fostering their academic success in Mathematics.

Challenges Encountered by the Students in Solving Linear Equations Using Cramer's and Paravartya's Rules

The study on the challenges encountered by students in solving systems of linear equations using Cramer's rule and Paravartya's rule revealed four main themes based on the responses of Grade 8 students: difficulty in navigating sign numbers, struggle in handling larger numbers, difficulty in transforming equations to standard form, and confusion in executing multi-step procedures. Each theme highlights distinct cognitive obstacles and areas where instructional strategies may need to be improved.

Theme 1. Difficulty in Navigating Signed Numbers. The first theme addresses the students' struggles with managing signed numbers, a recurring challenge throughout their application of Cramer's rule and Paravartya's rule. The difficulty in maintaining the correct signs during multi-step calculations is a significant barrier, leading to frequent calculation errors.

Several students noted that:

Those signs, sir. Those signs confuse me. SP 3

Yes, sir, I get confused with the signs. SP 4

I struggle with the signs, sir. It is just the signs I struggle with, sir (laughs). SP 7

This issue is common in mathematical problem-solving, as research has shown that students often struggle with the concept of signed numbers, especially when they are not consistently reinforced in earlier mathematical instruction (Bennett & Nelson, 2018). Research by Jones (2016) also highlighted those difficulties with signed numbers are prevalent in Algebra and linear equation solving, often leading to conceptual misunderstandings and procedural mistakes.

Theme 2. Struggle in Handling Larger Numbers. The second theme centers around the students' challenges in working with larger numbers, which can overwhelm them during multiplication and division. This is a common problem for learners, as larger numbers often increase cognitive load, making basic arithmetic operations more prone to errors.

As they said:

...and big numbers. SP3

Those very large numbers are when solving. Then, when you multiply them, the result becomes really big, which is why it is sometimes difficult. SP 5

There is something about very large numbers; I get confused about dividing them between two numbers or finding their smallest amount. SP 6

Sir, those big numbers take a long time to solve; that is why it takes me a while to finish. SP6

According to a study by Sweller et al. (2011), cognitive load theory suggests that complex computations with large numbers can exceed a student's working memory capacity, leading to mistakes

in operations. Additionally, his research indicates that students benefit from strategies that reduce cognitive load, such as breaking down larger numbers into more manageable parts or using estimation techniques to simplify calculations.

Theme 3. Difficulty in Transforming Equations to Standard Form. The third theme involves students' difficulties in transforming equations into standard form, which is essential in applying both Cramer's rule and Paravartya's rule.

Students revealed:

Those standard forms and large numbers. SP 3

I find it hard, sir, to convert the equation to standard form because I sometimes make mistakes. That is it, sir. SP 9

My struggle, sir, is really in transforming the equation—what you call converting to standard form. That is where I get stuck, sir, and that is why I cannot proceed with solving the answer. SP 10

This issue indicates a gap in students' foundational Algebraic knowledge, as earlier studies have shown that a lack of fluency in manipulating Algebraic expressions significantly hinders their ability to proceed with more advanced mathematical procedures (Lloyd et al., 2014). Furthermore, the ability to transform equations accurately is fundamental for solving systems of linear equations, and deficiencies in this area can lead to persistent errors in subsequent steps, affecting overall problem-solving performance.

Theme 4. Confusion in Executing Multi-Step Procedures. The fourth theme highlights confusion among students regarding the multi-step procedures involved in Cramer's rule and Paravartya's rule, especially in the cross-multiplication process.

Several students noted:

Well, sir, about cross multiplication. I get confused about how to multiply and what to do first, so it does not mix it up. SP 1

Well, sir, I get confused when multiplying, like transferring the numbers during multiplication. You might multiply the wrong thing first, leading to mistakes. SP 8

This complexity is not unusual, as multi-step problems often require a high degree of procedural knowledge and attention to detail. Research by Carpenter and Lehrer (2017) suggests that students often struggle with multi-step problem-solving because it requires procedural fluency and conceptual understanding. They argue that explicit instruction focusing on breaking down complex steps and reinforcing the relationships between different operations can help students manage these problems more effectively.

Thus, each theme reflects students' significant cognitive challenges in solving systems of linear equations using Cramer's rule and Paravartya's rule. Addressing these challenges through targeted instructional strategies, such as emphasizing signed number operations, providing practice with large numbers, reinforcing algebraic transformations, and clarifying multi-step procedures, can help improve students' problem-solving skills and increase their confidence in tackling such problems.

Students' Coping Strategies with the Challenges Encountered in Solving Linear Equations using Cramer's Rule and Paravartya's Rule

The strategies students use to cope with the challenges encountered in solving linear equations using Cramer's rule and Paravartya's rule reveal their reliance on a combination of memorization, chunking strategy, guided practice, and peer support. These coping mechanisms align with broader educational research, highlighting the importance of developing procedural fluency and conceptual understanding.

Theme 1. Memorization of the rules of signed numbers. The first challenge revealed that students have difficulty navigating signed numbers. This challenge poses a significant barrier to their success in more complex Mathematics in Cramer's rule and Paravartya's rule. This challenge also arises because students must recall multiple rules for adding, subtracting, multiplying, and dividing signed

numbers, often under time pressure or during multi-step calculations. The difficulty lies in remembering these rules and consistently applying them correctly, especially in more complex problems where signs can change multiple times.

The coping strategy the students should use, based on the result of the interview, revealed the significance of the first theme, *memorization of signed numbers*. Memorization plays a significant role in students' coping strategies, especially when facing complex mathematical tasks. Relying on memorization is a personal tactic and a recognized problem-solving method in mathematical learning.

Some participants said:

Well, I memorized (laughs) what to do first. What to do first in cross multiplication, and that is how I was able to answer, sir. SP 1

I studied, sir, the techniques for solving. SP 4

Well, sir, I memorized what to do first and the steps to follow. SP 87

Studies have shown that memorization can help students achieve short-term success by allowing them to follow structured procedures, especially when deeper understanding is not immediately attainable (Sweller, 1988). For instance, students often rely on rote learning for algorithms and step-based procedures like solving linear equations (Geary, 2006).

However, while memorization can facilitate problem-solving, it can also suggest a superficial understanding of the concepts, as students might be more focused on “how” to apply methods rather than “why” they work. This highlights the need for educators to connect procedural knowledge with conceptual understanding, ensuring that students can explain and justify their problem-solving approaches (Bransford et al., 2000).

While memorization is a coping mechanism, consistent practice often complements it to reinforce and deepen understanding. This leads to the next theme, where students demonstrate the importance of practicing key concepts to gain mastery and improve their problem-solving ability.

Theme 2. Chunking Strategies. The second theme, *chunking strategies*, which involve breaking down large numbers into smaller, more manageable parts for computation, has significantly enhanced mathematical understanding and efficiency in handling larger numbers. This coping strategy aligns with cognitive theories, particularly Miller's (1956) assertion that working memory capacity is optimized when information is grouped into smaller, coherent units. Furthermore, Sweller (1988) demonstrated that chunking reduces cognitive load, enabling learners to master complex mathematical processes more effectively.

Participants' reflections further illustrate the impact of chunking strategies on their mathematical development, as they noted:

I manually multiplied using the method taught by our elementary teacher on how to multiply large numbers. SP 5

What I do, sir, when the numbers are large, is break them down into smaller parts to solve them step by step until it becomes easier for me to solve, sir. SP 10

This highlights how foundational chunking techniques introduced in elementary education facilitated their comprehension of large-number multiplication. Moreover, this narrative illustrates how mastering sequential computational steps, a hallmark of chunking, increased their efficiency and reduced their struggles with complex computations.

Theme 3. Mastery or Guided Practice. The third challenge, which focused on the difficulty of transforming equations to standard form, poses a critical step in solving linear systems using methods like Cramer's rule and Paravartya's rule. When students were asked how they coped with this challenge, a theme of *mastery or guided practice* emerged. Students' reliance on practice to master Mathematical techniques is supported by research on skill development and expertise in Mathematics.

As they noted:

I studied the signs. SP 4

I mastered the first step of getting the number, and from there, I fully mastered it and experienced less difficulty. SP 6



*I kept checking and rechecking if I was doing the signs correctly to avoid mistakes in solving.SP7
...practiced. SP 8*

Repeated practice is crucial for students to internalize Mathematical procedures and increase their fluency in solving complex problems (Ericsson et al., 1993). Practicing steps, such as multiplication, in solving systems of linear equations reflects a common strategy for students to build confidence and reduce errors. Studies emphasize that mastery comes from repetition and focused practice, where students refine their technique over time.

Moreover, consistent practice improves procedural proficiency and enhances conceptual understanding, allowing students to recognize patterns and relationships within mathematical operations. This strategy promotes self-efficacy and empowers students to tackle increasingly difficult problems with less reliance on external aids (Bandura, 1997).

While mastery through practice is essential, students often encounter barriers that require additional support. As such, the next theme highlights the collaborative aspect of learning, where students turn to their peers for assistance and guidance.

Theme 4. Asking for Help from Peers. The last challenge focuses on students' confusion in executing multi-step procedures, often leading them to seek help from their peers. The theme “*Asking for Help from Peers*” emerged as the last coping strategy the students should use to encounter the last challenge. In an educational setting, collaboration and peer support are powerful strategies for overcoming learning challenges.

One student revealed:

I asked my classmate, sir. SP 3

also asked for help on how to do it. SP 3

I really struggled, sir. I got confused even though I studied the steps. It's a good thing my classmates were there; I could ask them for help and guidance. SP2

Students benefit from peer-assisted learning, as it provides opportunities for explanation, feedback, and the sharing of strategies. Peer interaction enables students to discuss and clarify concepts they might not fully grasp, promoting a deeper understanding and a more flexible application of learned methods (Vygotsky, 1978). Students' willingness to seek help from their peers reflects a social learning approach, where collaborative problem-solving is a central component of the learning process (Johnson & Johnson, 1994). Peer support also encourages a sense of community within the classroom, fostering motivation and reducing isolation when students struggle with difficult content (Roseth et al., 2008).

The themes of memorization, chunking strategy, mastery through practice, and seeking help from peers provide insight into students' strategies to solve linear equations using Cramer's and Paravartya's rules. Existing research supports each theme, highlighting the importance of individual and collaborative learning strategies. Educators can better support students in mastering complex mathematical techniques by addressing the balance between procedural fluency and conceptual understanding.

Learning Activity Sheet (LAS) that Enhances Students' Mathematics Achievement

The Learning Activity Sheet (LAS) in Mathematics 8 is a vital resource that aligns with the Department of Education's commitment to quality education. Created to meet the needs of Grade 8 learners, it aims to enhance understanding of key math concepts, build critical thinking, and promote problem-solving skills, establishing a foundation for advanced mathematics.

This LAS integrates interactive and structured activities that align with K-12 standards, allowing students to learn complex concepts progressively and at their own pace. It also connects lessons to real-life contexts, helping students see practical applications and develop positive attitudes toward math. Designed for both student engagement and educator support, this LAS is a collaborative tool that reinforces the Department of Education's dedication to inclusive and future-ready learning.



LAS 1: Graphing Systems of Linear Equations in Two Variables

This LAS focuses on understanding and analyzing systems of linear equations using graphical methods, which aid comprehension and retention. Graphing provides immediate feedback and fosters spatial reasoning, enhancing algebraic skills and conceptual understanding (Hall & Coles, 2012).

LAS 2: Solving Systems of Linear Equations in Two Variables

This LAS introduces advanced methods like Cramer's rule and Paravartya's rule for solving systems of equations. These techniques develop logical reasoning and problem-solving skills, offering essential tools for real-world applications in economics, engineering, and business (Markey, 2009).

LAS 3: Application of Systems of Linear Equations in Two Variables

This LAS highlights the practical use of solving systems of equations in real-world scenarios across business, economics, and science. Applying mathematical modeling fosters critical thinking and connects theory to real-life problems, motivating deeper engagement (Merck et al., 2020; Suh et al., 2015).

Learning Activity Sheet (LAS) that Enhances the Grade 8 Students' Mathematics Achievement

The Learning Activity Sheet (LAS) is recognized as an effective tool for enhancing students' academic achievement, particularly in Mathematics. LAS facilitates a deeper understanding and application of Mathematical concepts. This approach aligns with constructivist learning theories, which advocate for hands-on experiences and interactive learning to reinforce comprehension and skill development. Furthermore, Cheng and Tsai (2019) provide evidence that LAS is particularly effective in improving Algebraic problem-solving abilities. Their research demonstrates that engaging students in structured, problem-solving tasks fosters analytical thinking and enhances their ability to tackle complex Mathematical problems. These findings validate the positive impact of LAS in strengthening students' academic performance, especially in areas requiring higher-order thinking and problem-solving skills.

The identified challenges and coping strategies provide insights into addressing students' difficulties with Cramer's and Paravartya's rules. By integrating structured LAS activities and targeted instructional practices, educators can enhance conceptual understanding and procedural fluency, improving students' mathematical performance and confidence.

Discussion

This study focused on the effectiveness of Cramer's rule and Paravartya's rule in solving systems of linear equations in enhancing Mathematics achievement among Grade 8 students of the selected public school of Ozamiz City, Misamis Occidental, for the school year 2024-2025. The study evaluated the student's level of Mathematics achievement in solving systems of linear equations in the pretest and posttest, determined the significant differences between the pretest and posttest, and explored answers to these central questions: (1) What challenges do the students encounter when solving systems of linear equations using Cramer's rule and Paravartya's rule?, (2) How do the students cope with the challenges encountered in solving linear equations using Cramer's rule and Paravartya's rule?, and (3) What learning activity sheet involving solving linear equations using Cramer's rule and Paravartya's rule can be designed to uplift students' mathematics achievement?

Below is the summary of the findings:

A significant difference existed between the Grade 8 students' pretest and posttest scores using Cramer's rule ($p < 0.05$) and Paravartya's rule ($p < 0.05$), confirming its effectiveness in solving systems of linear equations. This means that Cramer's rule and Paravartya's rule are effective and engaging methods for solving systems of linear equations and improving Mathematics achievement. No significant difference existed between the posttest scores of Cramer's rules and Paravartya's rules, as confirmed in the p-value ($p < 0.05$) associated with a very small F-value ($F = 0.260$). This means that both methods are equally effective in solving systems of linear equations.





The challenges that the Grade 8 students encountered when solving systems of linear equations using Cramer's rule and Paravartya's rule were the following: Students struggled with signed numbers, operations on large numbers, transforming equations, and following procedural steps. Based on the result, the Grade 8 students coped with the challenges encountered in solving systems of linear equations using Cramer's rule and Paravartya's rule by using memorization, chunking, consistent practice, and peer collaboration to overcome difficulties. The Learning Activity Sheets in Mathematics 8 aimed to enhance understanding of key Math concepts, build critical thinking, and promote problem-solving skills to enhance the Grade 8 students' mathematics achievement.

The findings of this study highlight the effectiveness of Cramer's rule and Paravartya's rule in improving mathematics achievement among Grade 8 students. These results support the stated objectives by demonstrating measurable improvements in test scores and reinforcing the value of integrating alternative computational strategies into mathematics instruction. The discussion provides insights into how these findings align with existing educational frameworks and pedagogical practices.

Explicit connections to the theoretical framework, such as APOS theory, are particularly relevant. APOS theory emphasizes the importance of actions, processes, objects, and schemas in learning mathematics. Both methods leverage procedural fluency and conceptual understanding as students develop processes for solving equations and transform them into structured schemas. By aligning these results with APOS theory, the study underscores how students' learning pathways are enhanced through scaffolded methods like Cramer's and Paravartya's rules.

A notable observation is the absence of a significant difference between the posttest scores for both methods. This equivalence suggests that educators have flexibility in selecting either approach based on contextual needs rather than rigid effectiveness rankings. Classroom practices might, therefore, emphasize factors such as student preferences, resource availability, and learning objectives when deciding which method to adopt. Further exploration of these factors can inform adaptive teaching strategies.

Challenges identified in the study, such as difficulties with signed numbers and large computations, can be contextualized within cognitive load theory. High cognitive demands may hinder problem-solving efficiency, particularly for students still developing foundational mathematical skills. To address these issues, targeted interventions, such as using visual aids, breaking down complex steps, and integrating technology, could reduce cognitive overload and enhance comprehension.

Coping strategies, including memorization and peer collaboration, reflect principles of collaborative and scaffolded learning theories. These approaches not only mitigate difficulties but also encourage active engagement and knowledge construction. The benefits of collaborative learning, combined with guided practice, align with modern educational paradigms emphasizing inquiry-based and social learning models.

Implications for curriculum development and pedagogical practices are significant. Integrating both methods into instructional materials and teacher training programs can diversify problem-solving approaches and support differentiated instruction. The development of learning activity sheets tailored to address common challenges can further promote conceptual understanding and procedural fluency. Recommendations include embedding these strategies into professional development workshops and curriculum design initiatives to ensure broader applicability.

This study acknowledges its limitations, such as the small sample size and specific educational context, which may affect the generalizability of the findings. Future research could address these constraints by conducting studies in diverse settings with larger samples. Longitudinal research could explore the sustained impacts of these methods on students' mathematical performance. Additionally, investigating the integration of digital tools and visual aids could offer insights into enhancing the effectiveness of these methods. Research on the impact of teacher training and instructional delivery modes would also provide valuable perspectives for scaling implementation efforts.

Knowledge Contribution

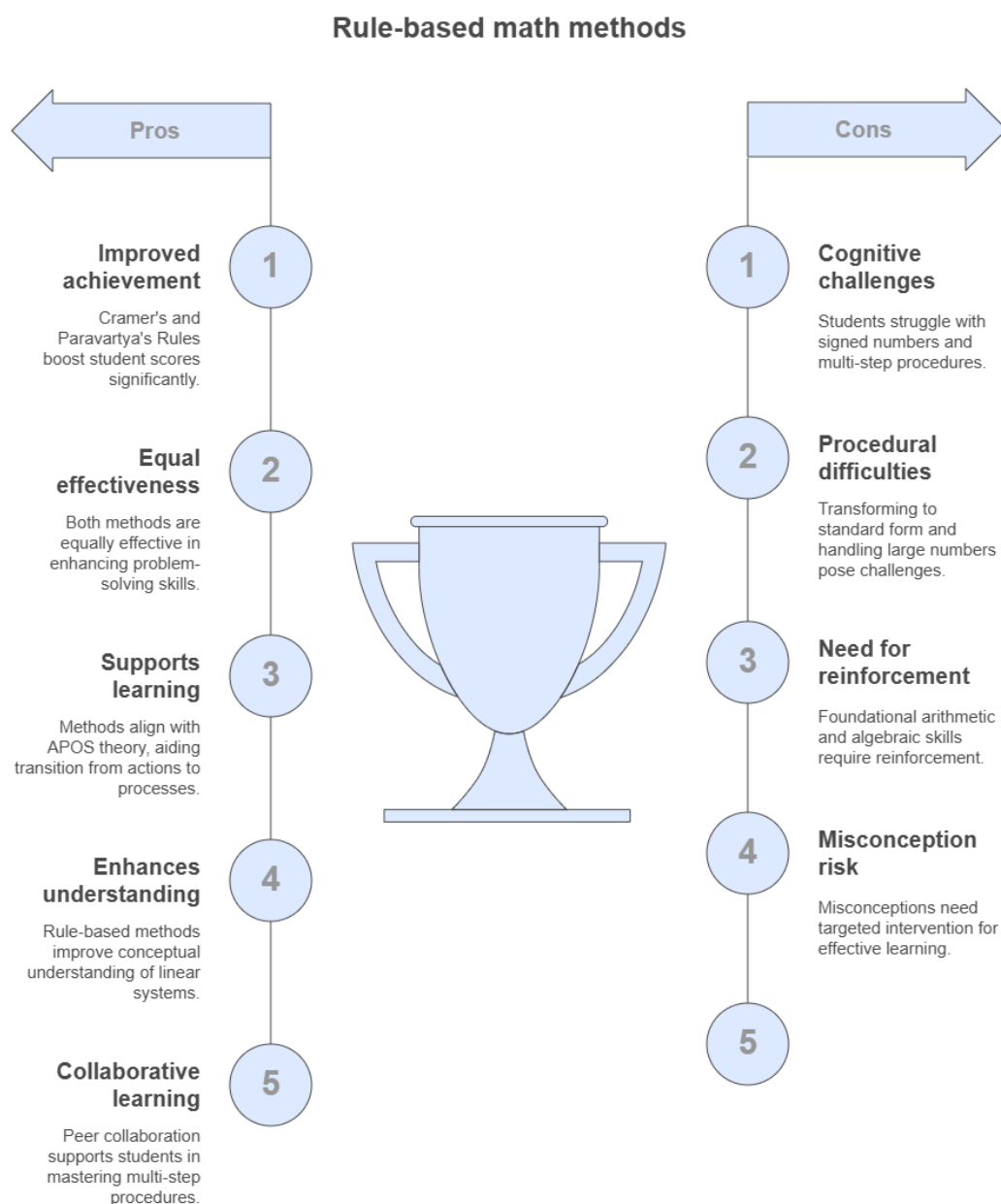


Figure 2 Concepts of Takeaways in Mathematics Instruction

The study found that students' performance in solving systems of linear equations can be improved by using both Cramer's Rule and Paravartya's Rule. The average scores for both groups increased significantly in the pretest and posttest results, with Paravartya's Rule producing a 61% gain and Cramer's Rule producing a 67% improvement. This suggests that both approaches assisted pupils in raising their math proficiency from below average to average. These findings demonstrate how using organized, rule-based approaches can significantly improve students' arithmetic comprehension and problem-solving skills.

It's interesting to note that there was no discernible difference in posttest performance between the two approaches when evaluating their efficacy. Both teaching methods are equally successful,

according to statistical testing (ANCOVA in particular), despite minor differences in mean scores. This implies that teachers can employ either approach with assurance because they are aware of the comparable advantages they offer. Furthermore, by demonstrating how both rules aid students in progressing from carrying out simple operations to implementing comprehensive procedures, this supports the tenets of the APOS (Action–Process–Object–Schema) paradigm.

The study did, however, also identify a number of difficulties that students encountered when learning these techniques. These included having trouble processing big numbers, putting equations into standard form, working with signed numbers, and following multi-step processes. These problems indicate procedural and cognitive deficiencies that may prevent pupils from using mathematical procedures to their fullest potential. These results highlight how crucial it is to strengthen fundamental algebraic and arithmetic abilities in order to facilitate more complex problem-solving.

Students used a range of coping mechanisms to deal with these difficulties. While some employed chunking techniques to simplify huge numbers, many depended on memorization to retain the steps. They became proficient via repetition and practice, and when they ran into problems, peer support was a great help. Students' resilience and willingness to participate in both independent and group learning are demonstrated by these adaptive behaviors. These observations can help educators design support networks that help students become more self-assured and independent.

The importance of Learning Activity Sheets (LAS), which offered organized, interactive exercises in line with curricular objectives, was also highlighted in the study. In addition to strengthening classroom instruction, these LAS tools helped students understand the significance of what they were studying by relating math lessons to actual circumstances. Students' comprehension was enhanced and various learning styles were accommodated by LAS's integration of rule-based processes and application assignments.

All things considered, the study emphasizes how crucial it is to blend cutting-edge techniques with fundamental teaching. Both Cramer's and Paravartya's rules provide efficient means of enhancing mathematical proficiency. Teachers should also encourage the development of both procedural fluency and conceptual understanding, as well as address typical student challenges, in order to have the most possible impact. Learner outcomes and mathematics instruction can be greatly improved by incorporating student-centered practices, encouraging collaborative learning, and integrating structured resources like LAS.

Recommendation

Based on the findings and conclusion of the study, the following are recommended:

Teachers may be encouraged to provide focused instruction to address challenges such as handling signed numbers and multi-step procedures. This may include strategies for working with larger numbers and transforming equations into standard form, which can help simplify the solution process for students. Teachers may consider providing a more structured approach when guiding students through applying Cramer's rule and Paravartya's rule. Detailed handouts, instructional videos, or step-by-step guides that break down the procedure with clear explanations and examples can ensure students follow the correct sequence of steps. Collaborative learning strategies, such as group work, peer tutoring, and problem-solving sessions, are recommended to promote active learning and peer support. These strategies allow students to clarify concepts, reinforce their understanding, and address challenges together, enhancing student learning and retention. Incorporating Cramer's rule and Paravartya's rule into the curriculum is advisable to expose students to diverse problem-solving techniques. This provides students with multiple approaches to solving systems of linear equations, catering to different learning styles and promoting flexibility in their mathematical thinking.

Future researchers may explore the effectiveness of combining Cramer's and Paravartya's rules with visual aids or digital tools. Offering concrete representations of the procedures and solutions could enhance students' conceptual understanding, particularly those struggling with abstract Algebraic concepts.



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